REACTION OF THE FERMION FIELD ON SPONTANEOUS CHANGE OF ITS MASS

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Abstract

Modification of the particles in the course of the source evolution is considered. Influence of this effect on multiplicities and correlations of the particles is displayed, including an enhancement of the production rates and identical particle correlations and also back-to-back particle-antiparticle correlations.

1 Introduction

It is well known that in quantum chromodynamics the chiral invariance is spontaneously broken at low energies and the fermion masses (say nucleon or constituent quark mass) arise essentially due to chiral simmetry breaking. In the course of phase transition at high fermion density or at high temperature the approximate chiral invariance is restored and the light (u, d, s) quark masses become small (ensuring small explicit violation of the symmetry). In practice the change of the fermion masses is treated (semi)classically. However it appears that, if the change of the mass is fast enough, then the quantum fermion field is changing in a specific way producing correlated fermion-antifermion pairs.

Similar effects were considered earlier for bosons (mesons [1, 2, 3, 4] and photons [5]) with application to heavy-ion collisions [5, 6]. Below we consider the case of spatially homogeneous large volume. Applications of this effect to heavy ion collisions will be given elsewhere.

Decomposition of the free fermion field is taken in the form:

$$\psi(x) = \int \frac{d^3k}{(2\pi)^{3/2}} e^{i\mathbf{kx}} \sum_{\nu=1}^{2} [u_\nu(\mathbf{k}) b_\nu(\mathbf{k}) e^{-iEt} + v_\nu(-\mathbf{k}) d_\nu^\dagger(-\mathbf{k}) e^{iEt}] \tag{1}$$

where $b_\nu, b_\nu^\dagger$ and $d_\nu, d_\nu^\dagger$ are annihilation and creation operators of particles and antiparticles, obeying standard anticommutation relations.

Bispinors $u_\nu(\mathbf{k}), v_\mu(\mathbf{k})$ are orthonormal, and bispinors $u_\nu(\mathbf{k})$ and $v_\mu(-\mathbf{k})$ having opposite momenta are orthogonal. For these bispinors related to Dirac equation

$$\left(i\gamma^0 \frac{\partial}{\partial t} - \gamma^n k^n - m \right) \psi(\mathbf{k}, t) = 0, \quad n = 1, 2, 3 \tag{2}$$

we take the standard representation

$$u_\nu(\mathbf{k}) = \frac{1}{N}(m + \gamma k) \begin{pmatrix} s_\nu \\ 0 \end{pmatrix}, \quad v_\nu(\mathbf{k}) = \frac{1}{N}(m - \gamma k) \begin{pmatrix} 0 \\ s_\nu \end{pmatrix} \tag{3}$$

with two-component unit spinors $(s_1)_\rho = \delta_{1\rho},\ (s_2)_\rho = \delta_{2\rho}$ and normalization factor

$$N \equiv N(E, m) = (2E(E + m))^{1/2} \tag{4}$$

2 Step-like variation of the fermion mass

Let the mass m in (5) depends on time, $m = m(t)$. We consider in this section the step-like variation of the mass. This simple case reveals the main features of the phenomenon. Let at time $t = 0$ the mass changes instantly from m_i to m_f. According to Dirac equation (2), the time derivative of the field ψ has jump discontinuity at $t = 0$. Therefore the field $\psi(\mathbf{k}, t)$ is continuous at the point $t = 0$,

$$\psi_i(\mathbf{k}, -\delta t) = \psi_f(\mathbf{k}, +\delta t), \quad \delta t \to \ 0 \tag{5}$$

Turning to decomposition (1) we choose the third coordinate axis along the direction of momentum $\mathbf{k}$:

$$k^{(3)} = k = |\mathbf{k}| \tag{6}$$

and use for u_ν, v_ν the representation (3). In this case the bispinors u_ν, v_ν correspond to definite helicities h ($h = 1/2$ for $\nu = 1$ and $h = -1/2$ for $\nu = 2$) and Eq.(1) gives decomposition over states with definite momentum and helicity. In view of continuity of the field ψ we have

$$u_{i,\nu}(\mathbf{k})b_{i,\nu}(\mathbf{k}) + v_{i,\nu}(-\mathbf{k})d^\dagger_{i,\nu}(-\mathbf{k}) = u_{f,\nu}(\mathbf{k})b_{f,\nu}(\mathbf{k}) + v_{f,\nu}(-\mathbf{k})d^\dagger_{f,\nu}(-\mathbf{k}) \tag{7}$$

(sum over ν). Multiplying both sides of (7) by $u^\dagger_{f,\mu}(\mathbf{k})$ and then by $v^\dagger_{f,\mu}(-\mathbf{k})$ and using orthogonality conditions we get the Bogoliubov transformation of the annihilation and creation operators expressing final-state operators through initial-state operators:

$$\begin{aligned} b_{f,\nu}(\mathbf{k}) &= \alpha_\nu(\mathbf{k})b_{i,\nu}(\mathbf{k}) + \beta_\nu(\mathbf{k})d^\dagger_{i,\nu}(-\mathbf{k}), \\ d^\dagger_{f,\nu}(-\mathbf{k}) &= -\beta_\nu(\mathbf{k})b_{i,\nu}(\mathbf{k}) + \alpha_\nu(\mathbf{k})d_{i,\nu}(-\mathbf{k}) \end{aligned} \tag{8}$$

for every $\nu, \mathbf{k}$, with coefficients

$$\begin{aligned} \alpha_\nu(\mathbf{k}) &= u^\dagger_{f,\nu}(\mathbf{k})u_{i,\nu}(\mathbf{k}) = v^\dagger_{f,\nu}(-\mathbf{k})v_{i,\nu}(-\mathbf{k}) = \frac{(E_f + m_f)(E_i + m_i) + k^2}{N_i N_f}, \\ \beta_\nu(\mathbf{k}) &= u^\dagger_{f,\nu}(\mathbf{k})v_{i,\nu}(-\mathbf{k}) = -v^\dagger_{f,\nu}(-\mathbf{k})u_{i,\nu}(\mathbf{k}) = \mp\frac{k(E_f + m_f - E_i - m_i)}{N_i N_f} \end{aligned} \tag{9}$$

for $\nu = 1, 2$ correspondingly where N_i, N_f are normalization factors of the spinors u_ν, v_ν in the initial and final states, given by (4). As can be seen from (9), the Bogoliubov coefficients $\alpha_\nu(\mathbf{k}), \beta_\nu(\mathbf{k})$ satisfy the condition:

$$|\alpha_\nu(\mathbf{k})|^2 + |\beta_\nu(\mathbf{k})|^2 = 1 \tag{10}$$

so that the transformation (8) is unitary SU(2)-transformation, as it must be to preserve the anticommutation relations. In the case (6) the Bogoliubov transformation connects creation and annihilation operators which have opposite directions of momenta and opposite spin projections (equal helicities). It can be shown that the last statement remains valid for any direction of the momentum $\mathbf{k}$. Let us note that for step-like transition the Bogoliubov coefficients (9) are real-valued. The coefficient $\beta_\nu(\mathbf{k})$ changes its sign if we change the sign of the momentum ($\mathbf{k} \to -\mathbf{k}$), or change $\nu(1 \leftrightarrow 2)$, or interchange the initial and the final states ($f \leftrightarrow i$).

In general the coefficients $\alpha_\nu(\mathbf{k}), \beta_\nu(\mathbf{k})$ of $SU(2)$-transformation (8) can be represented in the form

$$\alpha(\mathbf{k}) = \cos r(\mathbf{k}) e^{i\varphi_\alpha}, \qquad \beta(\mathbf{k}) = \sin r(\mathbf{k}) e^{i\varphi_\beta} \tag{11}$$

where

$$r(\mathbf{k}) = tan^{-1} |\beta/\alpha| \tag{12}$$

is the main evolution parameter and the phases $\varphi_\alpha, \varphi_\beta$ do not play important role and they will not be considered here. For step-like transition the phases are absent.

Using (6),(9) we get

$$\alpha^2(\mathbf{k}) = \frac{1}{2} + \frac{k^2 + m_i m_f}{2E_i E_f}, \qquad \beta^2(\mathbf{k}) = \frac{1}{2} - \frac{k^2 + m_i m_f}{2E_i E_f}, \tag{13}$$

As one can see from (13) the evolution parameter $r(\mathbf{k})$ is equal zero at $\mathbf{k} = 0$, it is maximal at

$$k^2 = k_m^2 = m_i m_f + \frac{m_i^2 m_f^2}{(m_i - m_f)^2} \tag{14}$$

and it falls down slowly at large k,

$$r(\mathbf{k}) \approx \frac{|m_i - m_f|}{2k} \tag{15}$$

In the point of the maximum the parameter r depends only on the ratio m_f/m_i and it is sizable if the mass ratio is small (or large), say $m_i \gg m_f$. In this case

$$\tan r = |\beta/\alpha| \approx 1 - 2\sqrt{\frac{m_f}{m_i}} \quad \text{for} \quad k^2 \approx m_i m_f \tag{16}$$

If one takes the fermion masses m_i and m_f to be the constituent quark mass($\sim$ 350 MeV) and the current quark mass ($\sim$ 5 MeV) then $\tan r_{max} \approx 0.76$ at $k \approx 42$ MeV.

In reality the change of the fermion mass has finite time duration τ and the step-like approximation is valid only if the momentum is much less than inverse time duration, $k\tau \ll 1$. The effect of finite time duration will be discussed in the next section.

3 Smooth transition

Let us consider the smooth variation of the fermion mass $m(t)$. In this case the coefficients $\alpha(\mathbf{k}), \beta(\mathbf{k})$ of the Bogoliubov transformation can be expressed through solutions of the Dirac equation (2) [8]. To solve the equation it is helpful to use its squared form, representing the Dirac ψ-function in (5) in the form

$$\psi(\mathbf{k}, t) = \left(i\gamma^0 \frac{d}{dt} - \vec{\gamma}\vec{k} + m(t) \right) \chi(\mathbf{k}, t) \tag{17}$$

Then the equation (2) takes the form:

$$\left(\frac{d^2}{dt^2} - i\gamma^0 \frac{dm}{dt} + k^2 + m^2(t) \right) \chi(\mathbf{k}, t) = 0 \tag{18}$$

which splits into two complex-conjugated equations for two upper and two lower components of χ. The bispinor χ can be written in the form

$$\chi_\nu(\mathbf{k}, t) = \begin{pmatrix} s_\nu \\ 0 \end{pmatrix} \varphi(\mathbf{k}, t) + \begin{pmatrix} 0 \\ s_\nu \end{pmatrix} \varphi(\mathbf{k}, t), \ \nu = 1, 2 \tag{19}$$

where s_ν are unit two-component spinors, see (3). As a result we have the second-order equation for scalar function $\varphi(\mathbf{k}, t)$:

$$\left(\frac{d^2}{dt^2} - i \frac{dm}{dt} + k^2 + m^2(t) \right) \varphi(\mathbf{k}, t) = 0 \tag{20}$$

which is nothing but oscillator equation with complex-valued variable frequency (energy).

Considering the smooth variation of the mass $m(t)$ we use the reference model with

$$m(t) = \frac{m_f + m_i}{2} + \left(\frac{m_f - m_i}{2} \right) tanh(2t/\tau) \tag{21}$$

for which (20) has the exact solution and contains the important parameter τ giving characteristic time of the mass variation. The resulting solution (17) of the classical Dirac equation (2) describes the evolution of the normalized fermion field [8]. Choosing one of the upper spinors s_ν (say s_1) we get

$$\begin{aligned} \psi(\mathbf{k},t) &= u_1(\mathbf{k},m_i,E_i)e^{-iE_it},\ t<0 \\ \psi(\mathbf{k},t) &= \alpha_1(\mathbf{k})u_1(\mathbf{k},m_f,E_f)e^{-iE_ft}+\beta_1(\mathbf{k})u_1(\mathbf{k},m_f,-E_f)e^{iE_ft},\ t>0 \end{aligned} \tag{22}$$

showing appearance of the negative-energy wave (creation of antiparticles) with correct Boboliubov coefficient [8]. The choice of another unit spinor in (22) gives similar result. The momentum $\mathbf{k}$ in (22) may have arbitrary direction. If we take the $\mathbf{k}$-direction along the third (spin quantization) axis then bispinors u_1 in (22) have definite helicity $h=1/2$ which is conserved in the course of transition. For the model (21) we get:

$$\left\{\begin{array}{c} |\alpha(\mathbf{k},\tau)|^2 \\ |\beta(\mathbf{k},\tau)|^2 \end{array}\right\} = \frac{\sinh\left(\frac{\pi\tau}{4}(\pm E_f+Ei-m_f+m_i)\right)\sinh\left(\frac{\pi\tau}{4}(E_f\pm Ei\pm m_f\mp m_i)\right)}{\sinh\left(\frac{\pi\tau}{2}E_f\right)\sinh\left(\frac{\pi\tau}{2}E_i\right)} \tag{23}$$

The Bogoliubov coefficients in (23) satisfy the crucial unitarity condition (10). In the limit of step-like transition they correspond to (9) as one can check by straightforward calculation.

As can be seen from (23), the coefficient $\beta(\mathbf{k},\tau)\to 0$ at $\mathbf{k}\to 0$ and it falls down for large $\mathbf{k}$:

$$|\beta(\mathbf{k},\tau)|\approx r(\mathbf{k},\tau)\approx\frac{\sinh\left(\pi\tau|m_i-m_f|/4\right)}{\sinh\left(\pi\tau k/2\right)},\ k\gg m_i,m_f \tag{24}$$

In the region $m_f\ll k\ll m_i$, where the coefficient $\beta(\mathbf{k})$ and the evolution parameter $r(\mathbf{k})$ are maximal for step-like transition, we get:

$$|\beta(\mathbf{k},\tau)|^2\approx\frac{1}{2}-\frac{1}{2}\left(\frac{m_f}{k}+\frac{k}{m_i}h(\pi m_i\tau/2)\right) \tag{25}$$

with

$$h(x)=\frac{1}{2}(1+x\coth x),\quad h>1 \tag{26}$$

where $m_i\gg m_f, k\tau\ll 1$ was taken in accordance with typical parameters $m_i\sim 350MeV, m_f\sim 5MeV, k\sim 40MeV$ in Section 2 and with typical time duration $\tau \sim 1fm$. As a result the maximum of $\beta(\mathbf{k},\tau)$ and $r(\mathbf{k},\tau)$ is reduced for finite τ and shifted to smaller momentum,

$$|\beta_{max}(\mathbf{k},\tau)|^2 \approx \frac{1}{2} - \sqrt{hm_f/m_i}, \quad k_m \approx \sqrt{m_i m_f/h} \tag{27}$$

due to h-factor. At large $k \gg \tau^{-1}$ the effect is exponentially small, $|\beta| \sim exp(-\pi k\tau/2)$ for smooth transition.

4 Fermion creation and their correlations

Using Bogoliubov transformation (17) one can find the final-state fermion momentum distributions and the final-state fermion correlations. We confine ourselves to symmetric case when the fermions with opposite momenta are produced in an equivalent way (say central collisions of identical nuclei). For simplicity the Bogoliubov coefficients will be taken to be real-valued and $k = |\mathbf{k}|$ dependent. We also consider the simple model – fast simultaneous transition of large homogeneous system at rest. The movement of the system can be taken into account by shifting of each moving element to its rest frame and integrating over proper times and space-time raidities of the elements of the system as it was done for photon production [5] in heavy ion collisions.

The final-state momentum distribution of the fermions (single-particle inclusive cross-sections) is given by (below we use notations b_ν, d_ν for operators having definite helicity $\nu = \pm 1/2$)

$$N_{f,\nu}(\mathbf{k}) = \frac{1}{\sigma}\frac{d\sigma_\nu}{d^3k} = \langle b^\dagger_{f,\nu}(\mathbf{k}) b_{f,\nu}(\mathbf{k})\rangle \tag{28}$$

where brackets mean averaging over initial state. Using (8) we get:

$$N_{f,\nu}(\mathbf{k}) = |\alpha(\mathbf{k})|^2 \langle b^\dagger_{i,\nu}(\mathbf{k}) b_{i,\nu}(\mathbf{k})\rangle - |\beta(\mathbf{k})|^2 \langle d^\dagger_{i,\nu}(-\mathbf{k}) d_{i,\nu}(-\mathbf{k})\rangle + |\beta(\mathbf{k})|^2 \frac{V}{(2\pi)^3} \tag{29}$$

and similar expression for antifermions (with interchange $b \leftrightarrow d$). The last term in *rhs* of (29) reflects the result of the vacuum rearrangement (the initial vacuum of fermions having mass m_i is not the ground state of b_f, d_f-operators). The factor $V/(2\pi)^3$ replaces $\delta^3(0)$ in the case of large but finite volume V.

It is convenient to introduce the level population function $n_\nu(\mathbf{k})$:

$$N_\nu(\mathbf{k}) = \frac{V}{(2\pi)^3} n_\nu(\mathbf{k}) \tag{30}$$

Then (29) and corresponding equation for antiparticles take the simple form (independently for every helicity)

$$n_{f,\nu}(\mathbf{k}) = |\alpha(\mathbf{k})|^2 n_{i,\nu}(\mathbf{k}) + |\beta(\mathbf{k})|^2 \left(1 - \bar{n}_{i,\nu}(-\mathbf{k})\right), \tag{31}$$

$$\bar{n}_{f,\nu}(\mathbf{k}) = |\alpha(\mathbf{k})|^2 \bar{n}_{i,\nu}(\mathbf{k}) + |\beta(\mathbf{k})|^2 \left(1 - n_{i,\nu}(-\mathbf{k})\right) \tag{32}$$

where the notation $\bar{n}$ is used for antifermions. In general one has to consider simultaneously the fermions and antifermions with momenta $\pm\mathbf{k}$ to get the full picture of the particle creation. For example, if there are no particles in the initial state then we have $|\beta(\mathbf{k})|^2$ particles of each kind in the final state. If there is one particle having momentum $\mathbf{k}$ and helicity ν in the initial state then we have

$$n_{f,\nu}(\mathbf{k}) = 1, \quad \bar{n}_{f,\nu}(-\mathbf{k}) = 0 \tag{33}$$

and $|\beta|^2$ particles in the rest of the states. If there is a fermion-antifermion (singlet) pair having zero momentum in the initial state, that is $n_{i,\nu}(\mathbf{k}) = 1,\ \bar{n}_{i,\nu}(-\mathbf{k}) = 1$, then we get

$$n_{f,\nu}(\mathbf{k}) = \bar{n}_{f,\nu}(-\mathbf{k}) = |\alpha(\mathbf{k})|^2, \quad n_{f,\nu}(-\mathbf{k}) = \bar{n}_{f,\nu}(\mathbf{k}) = |\beta(\mathbf{k})|^2 \tag{34}$$

for the final state. Therefore in zero-momentum states which are initially copletely occupied (both with fermions and antifermions) the occupation number decreases contrary to initially empty states where the occupation number increases. For termal equilibrium $n(\mathbf{k})$ is the Fermi distribution, depending on temperature and chemical potential and the both effects are present.

The transition effect is better seen in two-particle correlations. Two-particle inclusive cross-sections are given by

$$\frac{1}{\sigma}\frac{d^2\sigma^{++}_{\mu\nu}}{d^3k_1 d^3k_2} = \langle b^\dagger_{f,\nu}(\mathbf{k}_2) b^\dagger_{f,\mu}(\mathbf{k}_1) b_{f,\mu}(\mathbf{k}_1) b_{f,\nu}(\mathbf{k}_2)\rangle =$$
$$\langle b^\dagger_{f,\nu}(\mathbf{k}_2) b_{f,\nu}(\mathbf{k}_2)\rangle \langle b^\dagger_{f,\mu}(\mathbf{k}_1) b_{f,\mu}(\mathbf{k}_1)\rangle - \delta_{\mu\nu} \langle b^\dagger_{f,\nu}(\mathbf{k}_2) b_{f,\mu}(\mathbf{k}_1)\rangle \langle b^\dagger_{f,\mu}(\mathbf{k}_1) b_{f,\nu}(\mathbf{k}_2)\rangle \tag{35}$$

for two fermions with helicities μ, ν (correlation of identical fermions) and

$$\frac{1}{\sigma}\frac{d^2\sigma^{+-}_{\mu\nu}}{d^3k_1 d^3k_2} = \langle b^\dagger_{f,\nu}(\mathbf{k}_2) d^\dagger_{f,\mu}(\mathbf{k}_1) d_{f,\mu}(\mathbf{k}_1) b_{f,\nu}(\mathbf{k}_2)\rangle =$$
$$\langle b^\dagger_{f,\nu}(\mathbf{k}_2) b_{f,\nu}(\mathbf{k}_2)\rangle \langle d^\dagger_{f,\mu}(\mathbf{k}_1) d_{f,\mu}(\mathbf{k}_1)\rangle + \delta_{\mu\nu} \langle b^\dagger_{f,\nu}(\mathbf{k}_2) d^\dagger_{f,\mu}(\mathbf{k}_1)\rangle \langle d_{f,\mu}(\mathbf{k}_1) b_{f,\nu}(\mathbf{k}_2)\rangle \tag{36}$$

for fermion-antifermion pair. The last term in *rhs* of (36) is essential only in the presence of the time evolution of the fermion field. Here for simplicity we do not take into account the dynamical correlations arising due to Coulomb and strong interactions of the fermions.

Considering correlations in finite volume V we can use the modified creation and annihilation operators [1, 6] which are nonzero only in the volume V and satisfy modified anticommutation relations of the form

$$\left[b_\mu(\mathbf{k}_1), b^\dagger_\nu(\mathbf{k}_2)\right]_+ = \delta_{\mu\nu}\frac{V}{(2\pi)^3}G(\mathbf{k}_1-\mathbf{k}_2), \quad G(0)=1 \tag{37}$$

where $G(\mathbf{k})$ is the normalized form-factor of the fermion source (normalized Fourier transform of the source density). The corresponding correlator of the modified creation and annihilation operators has the form

$$\langle b^\dagger_\mu(\mathbf{k}_1), b_\nu(\mathbf{k}_2)\rangle \approx \delta_{\mu\nu}\frac{V}{(2\pi)^3}n((\mathbf{k}_1+\mathbf{k}_2)/2)G(\mathbf{k}_1-\mathbf{k}_2) \tag{38}$$

The right hand side of (37),(38) represents the smeared δ-function having the width of the order of the inverse size of the source which is much less than the width of the momentum distribution $n(\mathbf{k})$.

Using Bogoliubov transformation (8), substituting (38) for arising fermion correlators of initial-state operators and taking into account the small width of the form-factor $G(\mathbf{k})$ one can express the final-state correlators through Bogoliubov coefficients α, β and the form-factor G:

$$\begin{aligned}&\langle b^\dagger_{f,\nu}(\mathbf{k}_2)b_{f,\nu}(\mathbf{k}_1)\rangle\langle b^\dagger_{f,\nu}(\mathbf{k}_1)b_{f,\nu}(\mathbf{k}_2)\rangle \\ &= \frac{V^2}{(2\pi)^6}\left(|\alpha(\mathbf{k})|^2 n_{i,\nu}(\mathbf{k}) + |\beta(\mathbf{k})|^2(1-\bar{n}_{i,\nu}(\mathbf{k}))\right)^2 |G(\mathbf{k}_1-\mathbf{k}_2)|^2,\end{aligned} \tag{39}$$

$$\begin{aligned}&\langle b^\dagger_{f,\nu}(\mathbf{k}_2)d^\dagger_{f,\nu}(\mathbf{k}_1)\rangle\langle d_{f,\nu}(\mathbf{k}_1)b_{f,\nu}(\mathbf{k}_2)\rangle \\ &= \frac{V^2}{(2\pi)^6}|\alpha(\mathbf{k})\beta(\mathbf{k})(1-n_{i,\nu}(\mathbf{k})-\bar{n}_{i,\nu}(\mathbf{k}))G(\mathbf{k}_1+\mathbf{k}_2)|^2\end{aligned} \tag{40}$$

where smooth functions $\alpha(\mathbf{k}), \beta(\mathbf{k})$ can be evaluated at any of momenta $\mathbf{k}_1, \mathbf{k}_2 \approx \pm\mathbf{k}$ and we suggest that the process is $\mathbf{k} \leftrightarrow -\mathbf{k}$ symmetric. Taking sum over helicities ν we finally obtain the relative (divided by the product of single-particle distributions) correlation functions which are measured in experiment:

$$C^{++}(\mathbf{k}_1, \mathbf{k}_2) = 1 - \frac{1}{2}|G(\mathbf{k}_1 - \mathbf{k}_2)|^2, \tag{41}$$

$$C^{+-}(\mathbf{k}_1, \mathbf{k}_2) = 1 + \frac{1}{2}R^2(\mathbf{k})|G(\mathbf{k}_1 + \mathbf{k}_2)|^2 \tag{42}$$

with

$$R^2(\mathbf{k}) = \frac{|\alpha(\mathbf{k})\beta(\mathbf{k})(1 - n_i(\mathbf{k}) - \bar{n}_i(\mathbf{k}))|^2}{[|\alpha(\mathbf{k})|^2 n_i(\mathbf{k}) + |\beta(\mathbf{k})|^2(1 - \bar{n}_i(\mathbf{k}))]\,[|\alpha(\mathbf{k})|^2 \bar{n}_i(\mathbf{k}) + |\beta(\mathbf{k})|^2(1 - n_i(\mathbf{k}))]} \tag{43}$$

where we suggest that the helicities $\pm 1/2$ are equally represented in the initial state.

The equation (41) represents the identical particle correlation in its simplest (no interaction) form. The minus sign is a distinctive feature of the fermions and the factor $1/2$ arises due to sum over polarizations (only two of four final polarization states contain identical particles).

The equation (42) describes the time evolution effect which depends on evolution parameter r (or β, α). For the vacuum initial state the relative correlation function is big being equal to $\alpha^2/2\beta^2 > 1/2$. It is especially big for small β but in this case the pair production itself is weak, see (31),(32). In the case of cold dense matter the correlation function is big for empty (high momentum) initial states. In the case of hot quark-gluon plasma in the initial state one has to take into account the presence of effective termal quark mass which is much bigger than original quark mass ($m_i \sim gT$ for light quarks where g is QCD coupling constant and T is the temperature).

5 Conclusion

Calculation of the evolution effect for fermions shows that opposite-side fermion-antifermion correlations can be large. They can serve as a sign of chiral phase transition in quantum chromodynamics.

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